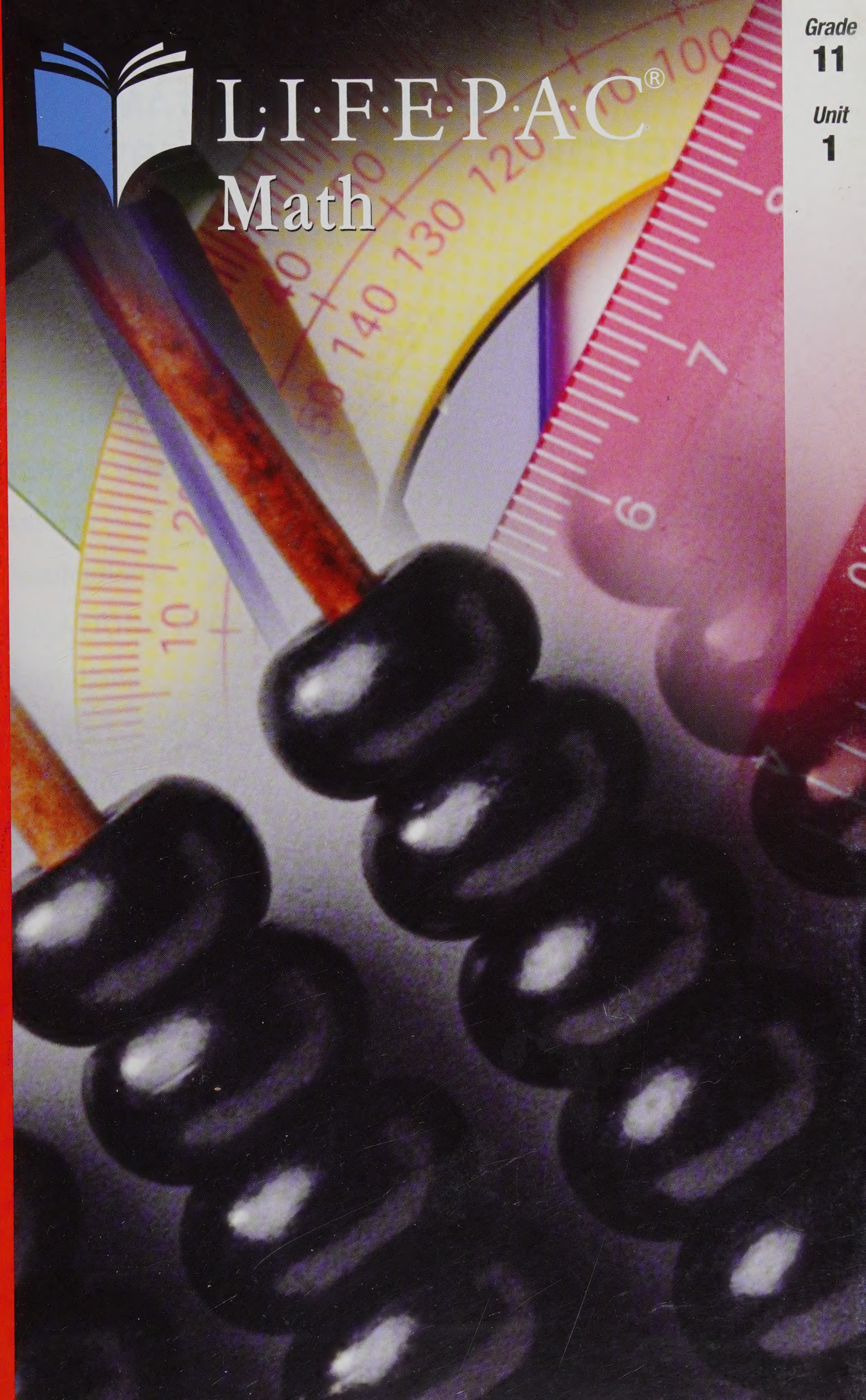
LIFEPAC® Math

Grade 11

Unit 1



Alpha Omega Publications®

MATHEMATICS 1101
SETS, STRUCTURE, AND FUNCTION

CONTENTS

Author: **Robert L. Zenor, M.A., M.S.**

Editor: Richard W. Wheeler, M.A.Ed.

Consulting Editor: Rudolph Moore, Ph.D.

Illustrator: Thomas Rush

Alpha Omega Publications®

804 N. 2nd Ave. E., Rock Rapids, IA 51246-1759

SETS, STRUCTURE, AND FUNCTIONS

The main theme in the study of mathematics in commonly understood to be the function. Importance of the function becomes more evident as you continue your study in mathematics. Many skills have to be mastered in order to study functions effectively. In addition to the concept of functions, this LIFEPAC® contains a study of sets and the algebra of sets, the properties of the real number system, the skills involved with combining algebraic terms, and the use of the exponent.

OBJECTIVES

Read these objectives. The objectives will tell you what you should be able to do when you have successfully completed this LIFEPAC.

When you have completed this LIFEPAC, you should be able to:

1. Combine sets through intersection and union.
2. Identify subsets.
3. Perform multiple operations of an expression in correct order.
4. Identify operational axioms when simplifying expressions.
5. Identify a function and its range and domain.
6. Evaluate functions.
7. Find the inverse of a function.
8. Express repetitive factors in exponential notation.
9. Evaluate numbers raised to a power.
10. Express fractions as numbers with negative exponents.
11. Simplify exponential notation.

Survey the LIFEPAC. Ask yourself some questions about this study. Write your questions here.

I. SETS

OBJECTIVES

When you have completed this section, you should be able to:

1. Combine sets through intersection and union.
2. Identify subsets.

Sets, a concept as old as man, has a definite place in the study of mathematics. From earliest times men have thought in terms of sets or collections of objects. We can think of such things as "sets of dishes," "members of a particular organization," or "the set of planets," as being natural in our environment. In mathematics, a set, symbolized { }, is a well-defined collection of objects. We need to define the properties of sets as they apply to mathematics so that we may use these properties to justify mathematical operations.

Natural numbers are counting numbers from +1 to infinity.

Whole numbers are all the natural numbers plus zero; whole numbers range from 0 to infinity.

Integers are all positive and negative numbers and zero; integers range from negative infinity to positive infinity.

REMEMBER?

PROPERTIES

In this section you will be concerned with the definitions, concepts, and operations of sets.

PROPERTIES OF SETS

A. Sets are made up of *members* or *elements*. The symbol used is $\in$.

B. A *finite* set is a set in which the number of elements is bounded by an interval. An *infinite* set is a set that is limitless.

C. Sets may be designated in one of two ways: the *list* method or the *rule* method. In the *list* method elements of the set are *listed*.

A. Model: If A is the set of whole numbers, then $7 \in A$ and is read, "Seven is a member of Set A."

B. Model 1: The set of states in the United States is finite and numbers 50.

Model 2: The set of whole numbers is infinite.

C. Model 1: If Set A is the set of even whole numbers less than 10, then the *list* method would be $A = \{0, 2, 4, 6, 8\}$.

Model 2: The *rule* method uses set-builder notation. $A = \{x \mid x \text{ is a whole number} < 10\}$. This notation is read, "A is the set of numbers such that x is a whole number less than 10." Note that 10 is *not* an element of the set.

n+1= Using the list method, write the following sets.

1.1 The odd integers between 2 and 15. ____________________

1.2 The even whole numbers less than 8. ____________________

1.3 The perfect square integers between 1 and 80 inclusive.

____

1.4 The last names of all your teachers.

____

1.5 Every two-digit number whose units' digit is three times its tens' digits.

____

n+1= Using the rule method, write the following sets.

1.6 $\{2, 4, 6, 8, 10, 12\}$ ____________________

1.7 $\{1, 8, 27, 64\}$ ____________________

1.8 $\{a, l, g, e, b, r\}$ ____________________

1.9 $\{2, 4, 6\}$ ____________________

1.10 $\{0\}$ ____________________

PROPERTIES OF SETS

D. Two sets are *equal* if the elements are identical.

E. Two sets are *equivalent* if they contain the same number of elements.

F. If every member of Set A is also a member of Set B, then A is a *subset* of B. This property in symbolic form is $A \subset B$.

G. An *empty* set is a set whose elements number zero. The symbol for an empty set is $\emptyset$.

H. The *empty* set is a subset of every set. This property in symbolic form is $\emptyset \subset A$.

D. Model: If $A = \{X, Y, Z\}$ and $B = \{Z, X, Y\}$, then $A = B$.

E. Model: If $A = \{X, Y, Z\}$ and $B = \{P, Q, R\}$, then A is *equivalent* to B.

F. Model: If $A = \{2, 4, 6\}$ and $B = \{1, 2, 3, 4, 5, 6\}$, then $A \subset B$.

Replace the question mark with the symbol = (equal to) or ≠ (not equal to) to make the expression true.

1.11 {2, 4, 6} ? {6, 4, 2} ________

1.12 $\{X, Y, Z\}$? $\{A, B, C\}$ ________

1.13 {0} ? { } ________

1.14 {1, 2, 3, 4, 5} ? {whole numbers between 1 and 5 inclusive} ________

1.15 {Adams, Polk, Harrison} ? {all Presidents of the United States} ________

Write the number of elements in each of the following sets.

1.16 $\{A, B, C\}$ ________

1.17 {0} ________

1.18 { } ________

1.19 {Students in your room} ________

1.20 {Whole numbers between 3 and 15} ________

Complete the missing part (?) that will make the statement true.

1.21 $8 \in \{3, 5, \underline{\ ?\ }\}$ ________

1.22 $\{6, 2\} \subset \{1, 2, 3, A, \underline{\ ?\ }\}$ ________

1.23 $\{1\} \underline{\ ?\ } \{5, 4, 3, 2, 1\}$ ________

1.24 $\{\ \} \underline{\ ?\ } \{0\}$ ________

1.25 $\{\ \} \underline{\ ?\ } \emptyset$ ________

Write the answers for the following three groups of problems.

Given that $A = \{a, b, c\}$

1.26 List all of the subsets of A that have exactly one element.

____

1.27 List all of the subsets of A that have exactly two elements.

____

1.28 List all of the subsets of A that have exactly three elements.

____

1.29 How many subsets does Set A have including $\emptyset$?

____

Given that $B = \{1, 2, 3, 4\}$

1.30 List all of the subsets of B that have exactly one element.

____

1.31 List all of the subsets of B that have exactly two elements.

____

1.32 List all of the subsets of B that have exactly three elements.

____

1.33 List all of the subsets of B that have exactly four elements.

____

1.34 How many subsets does B have including $\emptyset$?

____

Given $C = \{M, N, O, P, Q\}$

1.35 How many subsets does Set C have including $\emptyset$?

____

1.36 Write a formula for the number of subsets of any set; identify any symbols you use. ______________________

____

OPERATIONS

In arithmetic the operations, adding and multiplying, were defined and developed. Similarly, in the algebra of sets, we define two other operations. They are *union* and *intersection*. The algebra of sets is built around these two operations and around the properties just defined.

OPERATIONS OF SETS

A. The *union* of two sets, A and B, is a set whose elements are those that appear in *both* Set A and Set B, without repetition. The symbol for union is $\cup$.

B. The *intersection* of two sets, A and B, is a set whose elements are those that are *common* to A and B. The symbol for intersection is $\cap$.

A. Model 1: If $A = \{1, 2, 3\}$ and $B = \{2, 3, 4\}$, then $A \cup B = \{1, 2, 3, 4\}$. ($A \cup B$ is read the union of A and B.) Elements 2 and 3 are not repeated.

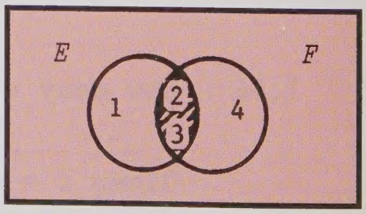
$A \cup B$ is commonly described as the elements of Set A *or* Set B in the sense that the elements of $A \cup B$--1, 2, 3, and 4--are found either in Set A *or* in Set B. The word *or* is used here not in the sense of choice.

Model 2: If $C = \{a, b, c, d\}$, and $D = \{a, c\}$, then $C \cup D = \{a, b, c, d\}$.

Notice that in Model 2 all the elements of D are in C. Therefore, the following property is evident:

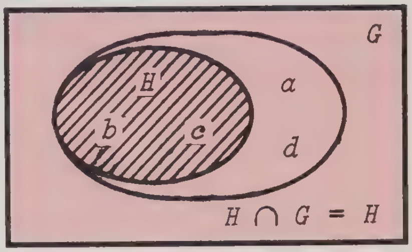
If $D \subset C$, then $C \cup D = C$.

B. Model 1: If $E = \{1, 2, 3\}$ and $F = \{2, 3, 4\}$, then $E \cap F = \{2, 3\}$. ($E \cap F$ is read the intersection of E and F.)



$E \cap F$ may be described also as the elements of Set E *and* Set F in the sense that the elements of $E \cap F$--2 and 3--are found in both Set E *and* in Set F. The word *and* is used here in the sense of common occurrence.

Model 2: If $G = \{a, b, c, d\}$ and $H = \{b, c\}$, then $G \cap H = \{b, c\}$.



Notice that in Model 2,

if $H \subset G$, then $H \cap G = H$.

Write the following sets.

Given $A = \{1, 2, 3, 4, 5\}$ $B = \{2, 4, 6\}$ $C = \{1, 3, 5\}$

1.37 $A \cup B$ ____________

1.38 $A \cup C$ ____________

1.39 $B \cup C$ ____________

1.40 $A \cap B$ ____________

1.41 $A \cap C$ ____________

1.42 $B \cap C$ ____________

1.43 $A \cup (B \cup C)$ ____________

1.44 $A \cap (B \cap C)$ ____________

1.45 $A \cup (B \cap C)$ ____________

1.46 $A \cap (B \cup C)$ ____________

Write the following sets.

Given $D = \{X \mid X$ is a whole number$\}$
$E = \{X \mid X$ is a perfect square $< 100\}$
$F = \{X \mid X$ is an even number between 10 and 20$\}$

1.47 $D \cup E$ ____________

1.48 $D \cup F$ ____________

1.49 $E \cup F$ ____________

1.50 $D \cap E$ ____________

1.51 $D \cap F$ ____________

1.52 $E \cap F$ ____________

1.53 $D \cup (E \cup F)$ ____________

1.54 $D \cap (E \cap F)$ ____________

1.55 $D \cup (E \cap F)$ ____________

1.56 $D \cap (E \cup F)$ ____________

Write the letter from Column II that matches Column I.

Column I		Column II
1.57 ________	If $A \subset B$	a. $\emptyset$
1.58 ________	If $B \subset A$	b. B
1.59 ________	$A \cup \emptyset$	c. Then $A \cup B = B$
1.60 ________	$A \cap \emptyset$	d. A
1.61 ________	$B \cup \emptyset$	e. Then $A \cap B = B$

Review the material in this section in preparation for the Self Test. The Self Test will check your mastery of this particular section. The items missed on this Self Test will indicate specific areas where restudy is needed for mastery.

SELF TEST 1

Write *true* or *false* (each question, 1 point).

1.01 ________ $A = \{X \mid X \text{ is an even number between 0 and 2}\} = \emptyset$.

1.02 ________ Set A is a subset of itself. $A \subset A$.

Write the complete answer on each line (each question, 5 points).

1.03 If $A \subset B$ and $B \subset A$, what else is true about sets A and B?

____

1.04 List *all* of the subsets of $\{a, b, c\}$ (No partial credit).

____

1.05 How many subsets are there of $\{a, b, c, d, e, f\}$? ________

Write the following unions and intersections (each answer, 5 points).

Given $A = \{1, 3, 5, 7, 9\}$, $B = \{2, 4, 6, 8, 10\}$, $C = \{1, 5, 6, 7, 9\}$

1.06 $A \cup B$ ________

1.07 $A \cap B$ ________

1.08 $A \cup (B \cap C)$ ________

1.09 $A \cap (B \cup C)$ ______________________

1.010 $A \cap B \cap C$ ______________________

34/42

Score ______________________
Teacher Check ______________________
Initial Date

II. STRUCTURE

OBJECTIVES

When you have completed this section, you should be able to:

3. Perform multiple operations of an expression in correct order.
4. Identify operational axioms when simplifying expressions.

This section contains axioms that will be used to justify the manipulation of algebraic expressions. *Axioms* are statements about mathematics that are accepted without proof. You might recall that *theorems* are statements about mathematics requiring proof. All the axioms or properties of the number system given in this section will be used in subsequent Algebra II LIFEPACs.

AXIOMS

In mathematics, addition and multiplication are *operations*. Subtraction and division are derived from these two operations. If we add or multiply two elements in a set, the result is a *unique* element of that set.

Closure: if $a \in R$ and $b \in R$, then $a + b$ and $a \cdot b$ are unique elements of R.

If a, b, $c \in R$, where R is a set of real numbers, then the following statements are true:

GENERAL PROPERTIES

A. *Reflexive property*: $a = a$.

B. *Symmetric property*: if $a = b$, then $b = a$.

C. *Transitive property*: if $a = b$ and $b = c$, then $a = c$.

PROPERTIES OF ADDITION

D. *Commutative*: $a + b = b + a$

E. *Associative*: $a + (b + c) = (a + b) + c$

F. *Identity*: $a + 0 = a$

G. *Additive inverse*: $a + (-a) = 0$

D. Models: $2 + 3 = 3 + 2$
$2 + x = x + 2$
$x + (3 + y) = x + (y + 3)$

E. Models: $3 + (5 + 2) = (3 + 5) + 2$
$4 + (2 + x) = (4 + 2) + x$
$(x + 4) + (y + 3) = (x + 4 + y) + 3$

F. Models: $6 + 0 = 6$
$x + 0 = x$
$(a + b) + 0 = a + b$

G. Models: $3 + (-3) = 0$
$x + (-x) = 0$

PROPERTIES OF MULTIPLICATION

H. *Commutative:* $a \cdot b = b \cdot a$

J. *Associative:* $a(b \cdot c) = (a \cdot b)c$

K. *Identity:* $a \cdot 1 = a$

L. *Multiplicative inverse:* $a \cdot \frac{1}{a} = 1;\ a \neq 0$

M. *Distributive:* $a(b + c) = (a \cdot b) + (a \cdot c)$

N. *Zero:* $a \cdot 0 = 0$

H. Models: $5 \cdot 7 = 7 \cdot 5$
$x \cdot 2 = 2 \cdot x$
$5(x + y) = (x + y)5$

J. Models: $10(5 \cdot 2) = (10 \cdot 5)2$
$10(x \cdot 3) = (10 \cdot x)3$

K. Models: $3 \cdot 1 = 3$
$x \cdot 1 = x$

L. Models: $5 \cdot \frac{1}{5} = 1$
$AB \cdot \frac{1}{AB} = 1;\ AB \neq 0$
$x \cdot \frac{1}{x} = 1;\ x \neq 0$

M. Models: $4(8 + 2) = (4 \cdot 8) + (4 \cdot 2)$
$3(5 - 2) = (3 \cdot 5) - (3 \cdot 2)$
$a(x + 2) = (a \cdot x) + (2 \cdot a)$ or $ax + 2a$

N. Models: $6 \cdot 0 = 0$
$x \cdot 0 = 0$
$(a - b)0 = 0$

Write the letter for the number property from Column II that justifies each statement in Column I.

		Column I		Column II
2.1	______	$6 + 2 = 2 + 6$	a.	reflexive
2.2	______	$8 = 8$	b.	symmetric
2.3	______	$\frac{1}{2} \cdot 2 = 1$	c.	transitive
2.4	______	$5 \cdot 0 = 0$	d.	commutative-addition
2.5	______	$6(3 \cdot 5) = (6 \cdot 3)5$	e.	commutative-multiplication
2.6	______	$4(8 + 1) = 4 \cdot 8 + 4 \cdot 1$	f.	associative-addition
2.7	______	$5 \cdot 2 = 2 \cdot 5$	g.	associative-multiplication
2.8	______	If $x = y$, then $y = x$	h.	distributive
2.9	______	$10 + (3 + 2) = (10 + 3) + 2$	i.	identity-addition
2.10	______	$6 + 0 = 6$	j.	identity-multiplication
2.11	______	$8 \cdot 1 = 8$	k.	multiplicative inverse
2.12	______	If $x = y$ and $y = 7$, then $x = 7$	l.	property of zero

APPLICATIONS

The properties or axioms are used to justify the operations of arithmetic--addition and multiplication. In this section we combine arithmetic expressions by addition and multiplication, and we justify each step by the number properties.

Write the number property(s) that justifies each of the following expressions.

		Property(s)
2.13	$6 + (14 + 70) = (6 + 14) + 70 = 20 + 70 = 90$	______
2.14	$8 + (6 + 0) = 8 + 6 = 14$	______
2.15	$2(5 \cdot 17) = (2 \cdot 5)17 = 10 \cdot 17 = 170$	______
2.16	$20 + (88 + 280) = 20 + (280 + 88) = (20 + 280) + 88 = 300 + 88 = 388$	______

2.17 $4 \cdot 13 \cdot 25 = 4 \cdot 25 \cdot 13 =$
$100 \cdot 13 = 1{,}300$

2.18 $8(\frac{1}{2} \cdot 17) = (8 \cdot \frac{1}{2}) \cdot 17 = 4 \cdot 17 = 68$

2.19 If *cba* represents three numbers multiplied together, what property allows you to rearrange the factors to read *abc*?

To multiply $6 \cdot 26$ mentally, rewrite 26 as $25 + 1$ and enclose in parentheses so that $6 \cdot 26 = 6(25 + 1) = 6 \cdot 25 + 6 \cdot 1 = 150 + 6 = 156$ (applying the *distributive* property). With some practice on paper, this manipulation can be done mentally.

Multiply the following expressions *mentally* and write the answer.

2.20 $8 \cdot 103 = 8(100 + 3) =$

2.21 $12 \cdot 22 = 12(20 + 2) =$

2.22 $9 \cdot 33 = 9(30 + 3) =$

2.23 $13 \cdot 12 = 13(10 + 2) =$

2.24 $17 \cdot 9 = 17(10 - 1) =$

2.25 $22 \cdot 23 = 22(20 + 3) =$

To find the value of a multi-step arithmetic expression, a definite order of operation must be followed.

Expressions without parentheses: Multiply and divide left to right *first*; then add and subtract.

Model:	$6 \cdot 2 \div 3 + 8 \cdot 4 \div 2$
First, multiply	$12 \div 3 + 32 \div 2$
then, divide	$4 + 16$
then, add	20

Expressions with parentheses: First, evaluate the numbers inside the parentheses; next, multiply and divide; then, add and subtract. If the parentheses contain more than one operation, evaluate the expression in parentheses in this order: multiply, divide, add, subtract. (Always work left to right)

Model 1:	$8(2 + 4) \div 16$
First, evaluate the expression in parentheses	$8(6) \div 16$
then, multiply	$48 \div 16$
then, divide	3

Model 2:	$20 - 5(8 - 6)$
First, evalute the expression in parentheses	$20 - 5(2)$
then, multiply	$20 - 10$
then, subtract	10

Evalute the following expressions.

2.26 $2 + 3 \cdot 6$ ______

2.27 $18 - 5 \cdot 3$ ______

2.28 $4 \cdot 6 + 3 \cdot 8$ ______

2.29 $6 \cdot 2 \div 6 + 1$ ______

2.30 $10 \div 5 + 6 \div 3$ ______

2.31 $5(4 \cdot 3 + 3 \cdot 4)$ ______

2.32 $8 - 2 \cdot 2$ ______

2.33 $(16 + 0)0$ ______

2.34 $\frac{1}{2} \cdot 8 + \frac{1}{3} \cdot 12 \div 2$ ______

2.35 $5(2 + 3) \div 25 + 8 \div 4$ ______

2.36 $3 + 4 \div 2 + 6(9 - 3) \div 12 + 1$ ______

2.37 $8 + [13 - (2 + 1)]$ ______

2.38 $5(5 + 2) - 2(5 - 4)$ ______

2.39 $4[(6 - 1) + 3(5 - 2)]$ ____________

2.40 $2[3 + 5(1 + 2)]$ ____________

Review the material in this section in preparation for the Self Test. This Self Test will check your mastery of this particular section as well as your knowledge of all previous sections.

SELF TEST 2

Complete these items.

2.01 Evaluate $10 + 4(3 + 2) + 5 + 12 \div 6$ (4 points).

____

2.02 What axiom allows $x10$ to be written as $10x$ (3 points)?

____

2.03 Name two axioms that allow $6 + (x + 5)$ to be written as $x + 11$ (3 points each). a. ____________ and b. ____________

2.04 Name the property that allows the statement $10 = y$ to be written as $y = 10$ (3 points). ____________

2.05 Evaluate $3\{5 + 3[10 + (4 \cdot 8)]\}$ (4 points).

____

Complete these items based on the following information (each answer, 3 points; except 2.06).

Given $A = \{1, 3, 5\}$, $B = \{2, 4, 6\}$, $C = \{1, 2, 3, 4, 5, 6, 7, 8\}$

2.06 Write all the subsets of Set A (each set, 1 point).

____

2.07 How many subsets are in Set C? ____________

2.08 Find $A \cup B$. ____________

2.09 Find $A \cap C$. ____________

2.010 Find $A \cap B \cap C$. ____________

31/37

Score ____________
Teacher Check ____________
Initial Date

III. RELATIONS AND FUNCTIONS

OBJECTIVES

When you have completed this section, you should be able to:

5. Identify a function and its range and domain.
6. Evaluate functions.
7. Find the inverse of a function.

Within mathematics are many sub-systems. These *systems* are generally independent of each other, yet they are inter-related and connected in several ways. You have studied one such system known as *Euclidean geometry*. You have also studied about *sets*. This section contains some of the basic ideas about *functions*. The terms and concepts of *range*, *domain*, *inverse*, and the *arithmetic of functions* help define this *system*.

DEFINITIONS

In this section you will study and apply the meanings of a relation and a function.

A *relation* is a set of ordered-pair numbers.

(6, 2) is an ordered-pair number where 6 is called the *first element* of the pair and 2 the *second element*. The pair (2, 6) is different from (6, 2) in that the order is different.

Model 1: $A = \{(6, 2), (2, 6), (4, 1), (4, 0)\}$

A is a *relation* because A is a set of ordered-pair numbers. A is *finite* with 4 members.

Model 2: $B = \{(x, y) \mid x + y = 5\}$

Set B is a *relation*; it is also an *infinite set*.

(Note that Set A is designated by the *list* method and Set B by the *rule* method. $A \cap B = \{(4, 1)\}$; $A \cup B$ is an infinite set.)

REMEMBER?

A *function* is a set of ordered-pair numbers such that for each first element there exists one and only one (unique) second element.

A function is a relation; however, all relations are not necessarily functions.

Model 1: $C = \{(2, 5), (6, 1), (8, 2), (9, 5), (4, 10)\}$

Set C is a function because each first element has a unique second element.

Model 2: $D = \{(2, 6), (5, 1), (2, 7), (8, 3)\}$

Set D is *not* a function because first element 2 has two different values for second elements, 6 *and* 7. *D is* a relation, however.

Model 3: $E = \{(x, y) \mid x - y = 8\}$

Set E is a function.

The set of first elements is called the *domain* set of the function.

Model: $F = \{(5, 1), (6, 2), (7, 3), (8, 4)\}$

The *domain* of $F = \{5, 6, 7, 8\}$

The set of second elements is called the *range* set of the function.

Model: $G = \{(-1, 3), (-2, 7), (-3, 9), (-4, 10)\}$

The *range* of $G = \{3, 7, 9, 10\}$

For sets A through F, write:
relation if the set is a relation but not a function;
function if the set is both a relation and a function; and
neither if the set is not a relation.

3.1 $A = \{1, 2, 3, 4, 5\}$ __________

3.2 $B = \{(1, 0), (0, 1)\}$ __________

3.3 $C = \{(2, 5), (2, 6), (2, 7)\}$ __________

3.4 $D = \{(1, 2), (2, 2), (3, 2), (4, 2)\}$ __________

3.5 $E = \{(3, 3), (4, 4), (5, 5), (6, 6)\}$ __________

3.6 $F = \{(x, y) \mid x + y = 10\}$ __________

Complete these activities.

3.7 Write the domain set of Problem 3.3. ________

3.8 Write the range set of Problem 3.5. ________

3.9 Write the range and domain of 3.6. ________

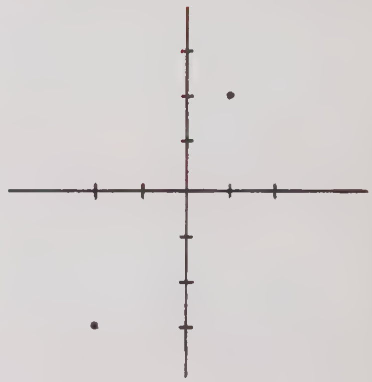
3.10 Write the range and domain of $P = \{(x, y) \mid y = x^2\}$. ________

GRAPHS

The graph of an ordered-pair number is a point on the rectangular coordinate axes. The first element is the measure of the distance along the horizontal axis. The second element is the measure of the distance on the vertical axis.

Model 1: The point A (1, 2) is located one unit to the right of 0, the origin, and two units up.

Model 2: The point B (-2, -3) is two units to the left of 0, the origin, and three units down.



Graph the following sets of points on the graph paper provided in this LIFEPAC. Label each graph with the letter of the corresponding set.

3.11 $A = \{(1, 2), (2, 3), (3, 4), (4, 5)\}$

3.12 $B = \{(1, 2), (2, 1), (3, 0), (4, -1)\}$

3.13 $C = \{(-1, 3), (0, 3), (1, 3), (2, 3)\}$

3.14 $D = \{(1, 1), (1, 2), (1, 3), (1, 4)\}$

3.15 $E = \{(6, 2), (7, 3), (6, -1), (5, 4)\}$

Complete these activities.

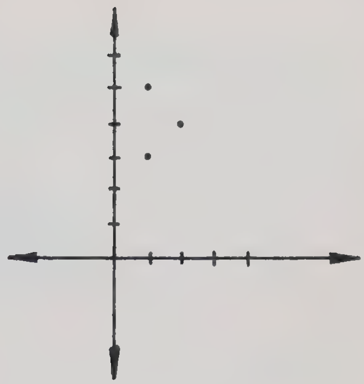
3.16 Which sets in Problems 3.11 through 3.15 are *not* functions?

____

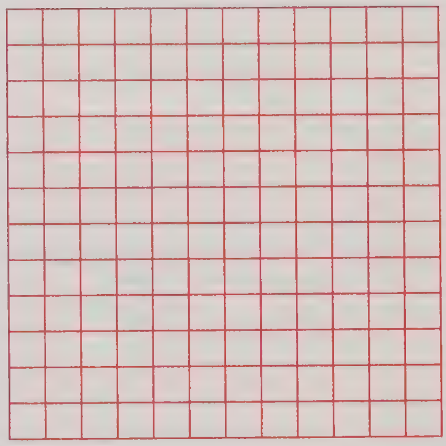
Notice that if you can draw a *vertical* line through two points of a set, the set is not a function.

Model: $F = \{(1, 3), (2, 4), (1, 5)\}$

(F is not a function.)



3.17 Find three members of the set $K = \{(x, y) \mid x - y = 5\}$, locate these points on the coordinate axis, and draw a line connecting the three points. Use $x = 0, 2, 4$.



3.18 Is Set K in Problem 3.17 a function? ____________

3.19 Is the graph in Figure 1 a function? ____________

3.20 Is the graph of the set of points in Figure 2 a function? ____________

Figure 1

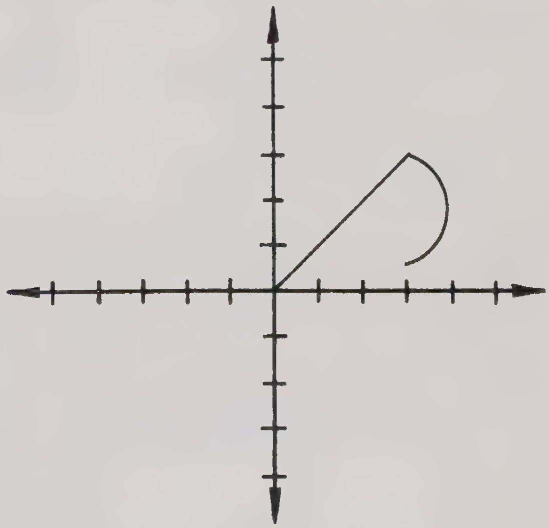


Figure 2

For convenience in this LIFEPAC and in subsequent LIFEPACs, we need to define a notation for a function. The graph of a set of ordered-pair numbers, for the most part, will be a line or a curve. This line or curve will be identified by the letter of the set. For example, $F = \{(x, y) \mid x - y = 1\}$ represents a line and it is also a function. We will call its graph, F. The graph of the function of $G = \{(x, y) \mid x + y = 1\}$ will be called G. The notation $F(x)$ is read F *of* x and denotes the graph F *at a particular value* of x. In Set F, if $x = 5$, then $y = 4$, because $5 - 4 = 1$. We use the notation that $F(5) = 4$. This notation means that when $x = 5$, the corresponding value of y in the F set is 4. $F(x)$ then becomes the vertical distance on a graph, or an expression for y. We often say, $y = F(x)$.

Model 1: $G = \{(x, y) \mid 2x + y = 10\}$.

To find $G(1)$, replace x with 1 in the rule for G and solve for the corresponding y value:

$2(1) + y = 10$ or $2 + y = 10$.
$y = 8$.

Thus, $G(1) = 8$, and we may write the ordered pair (1, 8) as an element of G.

Model 2: To find $G(-2)$, replace x with -2 in the rule for G (see Model 1) and solve for the corresponding y value:

$2(-2) + y = 10$ or $-4 + y = 10$.
$y = 14$.

Thus, $G(-2) = 14$, and the ordered pair is (-2, 14), a second element of G.

Model 3: $H = \{(x, y) \mid x - y = 2\}$.

Find $H(5) + H(2)$.

First, replace x with 5 in $x - y = 2$:

$5 - y = 2$. $y = 3$.

Thus, $H(5) = 3$.

MATHEMATICS

1101

LIFEPAC TEST

107/133

Name ____________________

Date ____________________

Score ____________________

MATHEMATICS 1101: LIFEPAC TEST

Complete these activities (each answer, 4 points).

Given $K = \{\bigcirc, \square, \triangle, \mathrm{I}\}$ $G = \{\triangle, \mathrm{I}, ▱\}$ $H = \{\square, ⊠, \otimes\}$

1. $K \cup G$ __________
2. $K \cap H$ __________
3. $K \cap G \cap H$ __________
4. All subsets of H __________
5. Number of subsets of $K \cup H$ __________
6. Number of elements of $K \cup G \cup H$ __________

Evaluate each expression (each answer, 3 points).

7. $12 + 8 \div 2 + 10$ __________
8. $15 \div 3 + 10 \div 2$ __________

Select the axiom(s) that could be used to justify the following statements. Choose from the following axioms: Multiplicative Inverse, Additive and Multiplicative Inverse, Commutative (addition and multiplication), Associative (addition and multiplication), or Distributive (each answer, 2 points).

9. $3 + (y + 5) = y + 8$ __________
10. $2 \cdot \frac{1}{2} = 1$ __________
11. $8 \cdot \frac{1}{8} + (-1) + 5 = 5$ __________
12. $10 + x \cdot 12 = 12x + 10$ __________

Complete these activities.

Given $f = \{(5, 1), (6, 2), (7, 3), (8, 1), (9, 7)\}$ (each answer, 3 points).

13. Write the range set of f __________
14. Write the domain set of f __________
15. Is f^{-1} a function? __________

Given $f(x) = x^2 + 5x$ and $g(x) = 2x + 1$ (each answer, 4 points).

16. $f(-2)$ __________
17. $g(3)$ __________

18. $f(5) + g(6)$ ________________

19. $g(3) - f(4)$ ________________

20. $g(a + h) - g(a)$ ________________

Write each expression in exponential notation (each answer, 3 points).

21. $a \cdot a \cdot a$ ________________

22. $3b \cdot 3b \cdot 3b \cdot 3b$ ________________

23. $2 \cdot a \cdot b \cdot a \cdot b$ ________________

Write each expression without exponents (each answer, 3 points).

24. $3x^2$ ________________

25. $(3a)^3$ ________________

26. $(abc)^2$ ________________

Express in simplified exponential notation (each answer, 3 points).

27. $10^2 \cdot 10^3$ ________________

28. $x^3 \cdot x^5$ ________________

29. $a^2 \cdot a^{-3} \cdot a$ ________________

30. $\dfrac{18a^3b^2}{2ab}$ ________________

31. $\dfrac{12ab^3c^2}{4a^{-2}bc^{-2}}$ ________________

Evaluate each expression (each answer, 3 points).

32. 2^3 ________________

33. 3^{-2} ________________

34. 7^0 ________________

Combine like terms (each answer, 4 points).

35. $5h - 2h$ ________________

36. $7a + 3a - 8a$ ________________

37. $6x^2 - 2x + x^2 + 5x$ ________________

38. $5(x - 2) + 3x$ ____________________

39. $7 - 2(5 - 2x)$ ____________________

40. $3(x + 2) + 4(x - 5)$ ____________________

NOTES

Then, replace x with 2:

$2 - y = 2. \quad y = 0.$

Thus, $H(2) = 0$.

Hence, $H(5) + H(2) = 3 + 0 = 3$.

Model 4: $F = \{(x, y) \mid 2x + y = 8\}$ and $G = \{(x, y) \mid y = x^2\}$.

Find $F(3) + G(2)$.

First, replace x with 3 in $2x + y = 8$:

$2(3) + y = 8$ or $6 + y = 8$.
$y = 2$.

Thus, $F(3) = 2$.

Then, replace x with 2 in $y = x^2$:

$y = 2^2. \quad y = 4.$

Thus, $G(2) = 4$.

Therefore, $F(3) + G(2) = 2 + 4 = 6$.

Previously we said that $y = F(x)$ or $y = G(x)$, depending upon the name of the set. A more convenient notation is to replace y with $f(x)$ in the rule of the set f. Hence, if $f = \{(x, y) \mid x + y = 10\}$, and if $y = f(x)$, we have

$$x + f(x) = 10 \text{ or } f(x) = 10 - x.$$

This form of the rule for set f is a more direct way of finding members of the set.

Model 5: If $f(x) = 10 - x$,

then $f(2) = 10 - 2$ or $f(2) = 8$,

and $f(20) = 10 - 20$ or $f(20) = -10$, and so on.

Evaluate the following functions.

Given that $f(x) = x^2 + 1$ and $g(x) = 3x + 1$

3.21 $f(2)$ ________ 3.24 $f(-6)$ ________

3.22 $f(6)$ ________ 3.25 $f(10)$ ________

3.23 $f(-1)$ ________ 3.26 $g(3)$ ________

3.27 $g(-2)$ __________

3.28 $g(0)$ __________

3.29 $g(-14)$ __________

3.30 $g(22)$ __________

3.31 $f(2) + g(3)$ __________

3.32 $f(5) - g(1)$ __________

3.33 $g(20) + f(6)$ __________

3.34 $[f(3) + g(-2)]^2$ __________

3.35 $f(-1) + 5g(-3)$ __________

3.36 $[f(2) + g(10)] \div 12$ __________

3.37 $2 \cdot f(4)$ __________

3.38 $3 \cdot g(5)$ __________

3.39 $[f(4)]^2$ __________

3.40 $[g(1)]^3$ __________

3.41 $2f(1) + 3g(4)$ __________

3.42 $[f(3)]^2 + g(5)$ __________

3.43 $[f(2) - g(1)]^2$ __________

3.44 $[f(6)]^2 - [g(3)]^2$ __________

3.45 $10f(101) - 10g(31)$ __________

Model 6: $A(r) = \pi r^2$. In this function, πr^2 represents the area of a circle; hence, A is the name of the set of ordered-pair numbers such that if r is known, then A can be found. For each r there exists a number called *area*. $A(2)$ means find the area of a circle given that the radius is 2. $A(2) = \pi 2^2 = 4\pi$.

Model 7: $h(x) = x^2 - 2x + 1$.

Find $h(a)$.

When proving theorems, many times we need to use constants such as a, b, and c in the h function. Hence, $h(a)$ means to find the corresponding second element when $x = a$:

$h(a) = a^2 - 2a + 1$; also,

$h(a + 1) = (a + 1)^2 - 2(a + 1) + 1$.

Evaluate the following functions.

Given $F(x) = 3x + 1$

3.46 $F(a)$ ____________________

3.47 $F(a + 1)$ ____________________

3.48 $F(a + h)$ ____________________

3.49 $F(a + h) - F(a)$ ____________________

3.50 $\dfrac{F(a + h) - F(a)}{h}$ ____________________

INVERSES

The inverse of a function is found by interchanging the range and domain and is designated by the symbol f^{-1}.

The *inverse* of a function is a set of ordered-pair numbers in which the range set is interchanged with the domain set.

Model 1: If $f = \{(2, 3), (5, 8), (7, 10)\}$, the *inverse* of f is the set $\{(3, 2), (8, 5), (10, 7)\}$. The symbol for the inverse set is f^{-1}. The -1 is not an exponent but the symbol for *inverse*.

Model 2: If $g = \{(1, 5), (2, 6), (3, 7), (4, 5)\}$, then $g^{-1} = \{(5, 1), (6, 2), (7, 3), (5, 4)\}$.

Notice in this example that g is a function, but g^{-1} is not. *In general*, if f is a function, f^{-1} is not necessarily a function.

Previously we said that $y = f(x)$. We then may substitute y for $f(x)$ in the equation $f(x) = 3x + 1$ to yield $y = 3x + 1$. To achieve the inverse of this function--that is, to interchange the *range*.

set for the *domain* set--first, interchange x and y in the equation $y = 3x + 1$; then solve for y. The new y is $f^{-1}(x)$.

Model 3: $y = 3x + 1$

Interchange: $x = 3y + 1$

Solve for y: $y = \frac{x - 1}{3}$

Thus, $f^{-1}(x) = \frac{x - 1}{3}$

Evaluate the following functions and inverses.

$f(x) = 3x + 1$ and $f^{-1}(x) = \frac{x - 1}{3}$

3.51 $f(2)$ = ____________ 3.54 $f^{-1}(10)$ = ____________

3.52 $f^{-1}(7)$ = ____________ 3.55 $f(0)$ = ____________

3.53 $f(3)$ = ____________ 3.56 $f^{-1}(1)$ = ____________

Write Problems 3.51 through 3.56 in ordered-pair form.

3.57 3.51 = (____ , ____) 3.60 3.54 = (____ , ____)

3.58 3.52 = (____ , ____) 3.61 3.55 = (____ , ____)

3.59 3.53 = (____ , ____) 3.62 3.56 = (____ , ____)

Answer this question.

3.63 Is the definition of *inverse* satisfied by these three pairs of answers in 3.57 through 3.62? ____________

Review the material in this section in preparation for the Self Test. This Self Test will check your mastery of this particular section as well as your knowledge of all previous sections.

SELF TEST 3

Complete these items.

3.01 Write the elements of Set A if $A = \{x \mid x$ is a positive integer less than 12$\}$ (4 points). ____________

3.02 Is Set A in Problem 3.01 finite or infinite (2 points)?

____

3.03 If $B = \{1, 2, 3, 4, 5\}$ and $C = \{3, 4, 5, 6, 7\}$, find (3 points each)

a. $B \cap C$ _______________

b. $B \cup C$ _______________

3.04 If $P = \{A, B, C, D\}$, list all of the subsets of P (4 points).

____

3.05 How many elements are in Set $C = \{(1, 2), (3, 4), (5, 6)\}$

(1 point)? _______________

Write *true* or *false* (2 points each).

$J = \{2, 4, 6, 8\}$, $K = \{1, 2, 3, 4, 5\}$, $L = \{1, 2, 3, 4, 5, 6, 7, 8\}$

3.06 _______________ $3 \in K$

3.07 _______________ $K \subset J$

3.08 _______________ $J \subset L$

3.09 _______________ $J \cup K = L$

3.010 _______________ $J \cup L = L$

Write the property that allows the left member to be changed to the right member (3 points each).

3.011 $b + a = a + b$ _______________

3.012 $a(b + c) = ab + ac$ _______________

3.013 $3 + (2 + 5) = (3 + 2) + 5$ _______________

3.014 $8 + 0 = 8$ _______________

3.015 If $2 = x$, then $x = 2$ _______________

Evaluate the following expressions (5 points each).

3.016 $3 + 2 \cdot 8 \div 4$ _______________

3.017 $[3(5 + 6) + 2] \div 7$ _______________

Given $F(x) = x^2 + 2$, $G(x) = 3x + 1$, $H(x) = x$

3.018 $F(2) =$ _______________

3.019 $G(-1) =$ _______________

3.020 $H(5)$ ______

3.021 $F(1) + F(5)$ ______

3.022 $F(2) - F(8) + G(1)$ ______

3.023 $F(a) + G(a) + H(a)$ ______

Complete the following activities (5 points each).

Given $P = \{(1, 2), (2, 2), (3, 3), (4, 3), (5, 7)\}$

3.024 the domain set = ______

3.025 the range set = ______

3.026 P^{-1} = ______

78/97

Score ______
Teacher Check ______
Initial Date

IV. ALGEBRAIC EXPRESSIONS

OBJECTIVES

When you have completed this section, you should be able to:

8. Express repetitive factors in exponential notation.
9. Evaluate numbers raised to a power.
10. Express fractions as words with negative numbers.
11. Simplify exponential notation.

Mathematical language involves the use of the variable. When variables are connected with the operational signs +, -, x, and — or ÷, we have an *algebraic expression*. Over five hundred years before Christ, Pythagoras knew of the relationship between the legs of a right triangle and its hypotenuse. Many years later the theorem known as the Pythagorean Theorem was translated into mathematical symbols. The algebraic expression of this theorem is $a^2 + b^2 = c^2$, where a and b represent the legs and c represents the hypotenuse.

In this section you will review the fundamental skills involved in working with the algebraic terms that make up algebraic expressions.

EXPONENTS

Exponents are shorthand symbols used to simplify an expression. Specific rules govern their use in mathematical operations.

DEFINITION

The *exponent* is a small-sized number written above and to the right of a term which serves to count the number of times that a base is used as a factor.

In the expression a^2 (shorthand for $a \cdot a$), a is the *base* and the exponent, or power, is 2.

Thus $5 \cdot 5 \cdot 5 = 5^3$ and $10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 10^5$. A base with an exponent is in *exponential notation*, in which we say we raise the base to a *power*.

Model 1: Express the following operations in exponential notation.

$8 \cdot 8 \cdot 8 = 8^3$

$x \cdot x \cdot x \cdot x = x^4$

$5p \cdot 5p = (5p)^2$

Model 2: Rewrite each operation without exponents.

$4^3 = 4 \cdot 4 \cdot 4 = 64$

$(7x)^2 = 7x \cdot 7x$

$(abc)^2 = abc \cdot abc$

Model 3: Simplify.

$(3x)^2 = 3x \cdot 3x = 9x^2$

$(5a)^3 = 5a \cdot 5a \cdot 5a = 125a^3$

For any number a, $a^1 = a$.

Models: $5^1 = 5$

$(2a)^1 = 2a$

$(-8)^1 = -8$

For any number a (except zero), $a^0 = 1$.

Models: $5^0 = 1$

$4^0 = 1$

$(3x)^0 = 1$

The following pattern of exponential numbers suggests a meaning for the negative exponent.

$10^3 = 1000$	$10^{-1} = ?$
$10^2 = 100$	$10^{-2} = ?$
$10^1 = 10$	$10^{-3} = ?$
$10^0 = 1$	

To complete the pattern,

$$10^{-1} = \frac{1}{10},$$

$$10^{-2} = \frac{1}{100}, \text{ and}$$

$$10^{-3} = \frac{1}{1000}.$$

For any nonzero number a, and any integer n, $a^{-n} = \frac{1}{a^n}$.

Models: $5^{-1} = \frac{1}{5}$

$8^{-2} = \frac{1}{8^2}$

$(2x)^{-3} = \frac{1}{(2x)^3}$

Model: Simplify 4^{-3}.

$4^{-3} = \frac{1}{4^3} = \frac{1}{4 \cdot 4 \cdot 4} = \frac{1}{64}$

Write in exponential notation.

4.1 $6 \cdot 6 \cdot 6$ __________

4.2 $7 \cdot 7 \cdot 7 \cdot 7$ __________

4.3 $x \cdot x$ __________

4.4 $3a \cdot 3a \cdot 3a$ __________

4.5 $r \cdot r \cdot r \cdot r \cdot r$ __________

4.6 $(8y)(8y)(8y)$ __________

4.7 $5 \cdot 5 \cdot 5 \cdot b \cdot b \cdot b \cdot b$ __________

4.8 $10 \cdot 10 \cdot a \cdot a \cdot b$ __________

4.9 $4 \cdot 4 \cdot 4 \cdot a \cdot b \cdot a \cdot b$ __________

Write without exponents, but *do not* evaluate.

4.10 5^2 __________

4.11 $(-2)^3$ __________

4.12 $(ab)^4$ __________

4.13 10^4 __________

4.14 $(-a)^3$ __________

4.15 $(2c)^5$ __________

4.16 $(3x)^2$ __________

4.17 $(-3x)^4$ __________

4.18 8^3 __________

Evaluate each of the following.

4.19	2^0	__________	4.25	$\frac{1}{10^0}$	__________
4.20	6^{-1}	__________	4.26	$(xy)^0$	__________
4.21	6^{-3}	__________	4.27	$(5x)^0$	__________
4.22	$(a + b)^0$	__________	4.28	2^{-2}	__________
4.23	3^1	__________	4.29	$\frac{5^{-1}}{5^0}$	__________
4.24	8^{-1}	__________	4.30	$(-7)^{-1}$	__________

Rewrite, using a negative exponent.

4.31	$\frac{1}{3^2}$	__________	4.34	$\frac{1}{8^6}$	__________
4.32	$\frac{1}{(xy)^{+1}}$	__________	4.35	$\frac{5}{x^2}$	__________
4.33	$\frac{1}{a^4}$	__________	4.36	$\frac{1}{16^2}$	__________

MULTIPLICATION AND DIVISION

By the definition of exponents, a^3 means a multiplied by itself three times, or $a \cdot a \cdot a$; and a^2 means a multiplied by itself twice, or $a \cdot a$.

Multiplication and division of bases with exponents involves adding and subtracting the exponents.

$$a^3 \cdot a^2 = (a \cdot a \cdot a)(a \cdot a)$$
$$= a^5$$

This example suggests that in *multiplying* like bases, the exponents are *added.*

$$\frac{a^5}{a^3} = \frac{a \cdot a \cdot a \cdot a \cdot a}{a \cdot a \cdot a}$$
$$= a^2 \text{ (three of the factors reduce)}$$

Likewise, *division* by exponential numbers of like bases results in *subtracting* their exponents.

THEOREMS

A. $a^m \bullet a^n = a^{m+n}$

B. $a^m \div a^n = a^{m-n}$

A. Models: $6^2 \cdot 6^3 = 6^{2+3} = 6^5$

$10^3 \cdot 10^5 = 10^{3+5} = 10^8$

$x^4 \cdot x^5 = x^{4+5} = x^9$

B. Models: $\dfrac{x^7}{x^3} = x^{7-3} = x^4$

$\dfrac{(5p)^8}{(5p)^3} = (5p)^{8-3} = (5p)^5$

Perform the indicated operations.

4.37 $x^2 \cdot x^5$ ____________

4.38 $r^2 \cdot r \cdot r^5$ ____________

4.39 $\dfrac{x^{10}}{x^4}$ ____________

4.40 $\dfrac{(a + b)^9}{(a + b)^4}$ ____________

4.41 $\dfrac{10x^5}{2x^2}$ ____________

4.42 $\dfrac{3.6x^5}{1.2x^2}$ ____________

4.43 $y^7 \cdot y^9$ ____________

4.44 $2x \cdot 3x^2$ ____________

4.45 $\dfrac{a^5}{a}$ ____________

4.46 $\dfrac{(3x)^{12}}{(3x)^4}$ ____________

4.47 $\dfrac{15a^4}{3a^2}$ ____________

4.48 $\dfrac{0.105p^6}{0.5p^3}$ ____________

4.49 $p^3 \cdot p^2 \cdot p$ ____________

4.50 $5a^2 \cdot 6a^4$ ____________

4.51 $\dfrac{b^7}{b^6}$ ____________

4.52 $\dfrac{a^5}{a^3}$ ____________

4.53 $\dfrac{20b^2c^3}{4bc}$ ____________

4.54 $\dfrac{7.2q^4r^5}{.6q^3r^3}$ ____________

Models: $x^5 \cdot x^{-3} = x^{5+(-3)} = x^2$

$$p^{-4} \cdot p^{-5} = p^{(-4)+(-5)} = p^{-9} = \frac{1}{p^9}$$

$$\frac{x^5}{x^{-3}} = x^{5-(-3)} = x^8$$

$$\frac{y^{-7}}{y^{-3}} = y^{(-7)-(-3)} = y^{-4} = \frac{1}{y^4}$$

Multiply or divide as indicated.

4.55 $x^4 \cdot x^{-2}$ ________

4.56 $p^8 \cdot p^{-3} \cdot p^2$ ________

4.57 $\dfrac{x^5}{x^{-3}}$ ________

4.58 $\dfrac{a^3b^2}{a^{-1}b^{-3}}$ ________

4.59 $x^{-3} \cdot x^7$ ________

4.60 $y^{-9} \cdot y^{-8} \cdot y^{10}$ ________

4.61 $\dfrac{x^{-8}}{x^{-7}}$ ________

4.62 $\dfrac{p^{-4}q^5r^6}{p^{-3}qr^{-2}}$ ________

4.63 $x^{-8} \cdot x^{-2}$ ________

4.64 $b^3 \cdot b^{-3}$ ________

4.65 $\dfrac{x^{-8}}{x^5}$ ________

4.66 $\dfrac{10}{10^{-2}}$ ________

EXPONENTS OF EXPONENTIAL EXPRESSIONS

Exponential expressions can be simplified in the following manner:

Simplify $(x^3)^4$.

By definition of exponential expressions,

$$(x^3)^4 = x^3 \cdot x^3 \cdot x^3 \cdot x^3 = x^{12}.$$

Also, $(p^2)^5 = p^2 \cdot p^2 \cdot p^2 \cdot p^2 \cdot p^2 = p^{10}$.

The previous examples suggest the following theorem.

THEOREM $(x^a)^b = x^{ab}$

Models: Simplify each of the following expressions:

$(x^3)^3 = x^9$

$(p^2)^5 = p^{10}$

$(a^2b^2)^4 = a^8b^8$

$(x^3)^{-2} = x^{-6} = \frac{1}{x^6}$

Simplify each of the following expressions *and* evaluate where possible.

4.67 $(x^2)^4$ ____

4.68 $(r^3)^{-2}$ ____

4.69 $(a^2b^2)^3$ ____

4.70 $(6^3)^{-3}$ ____

4.71 $(x^2y^3z^4)^2$ ____

4.72 $(2a^2)^3$ ____

4.73 $(-5x^2)^3$ ____

4.74 $(x^2)^5$ ____

4.75 $(s^{-3})^3$ ____

4.76 $(x^3y^2)^4$ ____

4.77 $\frac{1}{(x^4)^{-2}}$ ____

4.78 $(ab^2c^3)^{-2}$ ____

4.79 $(3b^3)^4$ ____

4.80 $(-4x^2)^2$ ____

4.81 $(p^4)^2$ ____

4.82 $(t^9)^{-8}$ ____

4.83 $(x^4y^2)^3$ ____

4.84 $\frac{1}{(y^{-6})^2}$ ____

4.85 $(r^{-3}s^2t)^4$ ____

4.86 $(4c^2)^3$ ____

4.87 $(3a^2b)^2$ ____

____

COMBINING TERMS

Algebraic expressions will need to be simplified as much as possible. One such simplification process is to combine terms by addition or subtraction. The *distributive property* also gives us the means by which terms may be combined.

The distributive property for real numbers a, b, and c follows this pattern:

$$ab + ac = a(b + c) \text{ and}$$

$$ab - ac = a(b - c).$$

Notice that a is a common factor on the left side of each equation; therefore, a may be "factored out" or set as the *coefficient* of $(b + c)$ or $(b - c)$.

Models:

$$3x + 5x = (3 + 5)x = 8x$$

$$2z + 7z = (2 + 7)z = 9z$$

$$9k - 3k = (9 - 3)k = 6k$$

$$\begin{aligned} 7y + 3y - 2y &= (7 + 3)y - 2y \\ &= 10y - 2y \\ &= (10 - 2)y \\ &= 8y \end{aligned}$$

In the expression $7x + 6y$, there is *no common factor*; therefore, the terms cannot be combined.

Model 1:

$$\begin{aligned} 5x + 2y - 3x &= 5x - 3x + 2y \\ &= (5 - 3)x + 2y \\ &= 2x + 2y \end{aligned}$$

$2x + 2y$ is the *simplist form* of $5x + 2y - 3x$.

Model 2: Simplify $6x^2 + 2x^2 - 3x^2 + y$.

$$\begin{aligned} 6x^2 + 2x^2 - 3x^2 + y &= (6 + 2)x^2 - 3x^2 + y \\ &= 8x^2 - 3x^2 + y \\ &= (8 - 3)x^2 + y \\ &= 5x^2 + y \end{aligned}$$

Simplify when possible.

4.88 $5x + 3x$ ________

4.89 $8x^2 - 2x^2$ ________

4.90 $3a + 3a - 2a$ ________

4.91 $10x^2 - y + x^2$ ________

4.92 $2x + 3x + 4y + 5y$ ________

4.93 $7x - 4x$ ________

4.94 $10b^3 - 4b^3$ ________

4.95 $12a + 16a - 20a$ ________

4.96 $3x + 2y - y$ ________

4.97 $7x + 2x + 5y - 2y$ ________

4.98 $10a + 13a$ ________

4.99 $14x - 12x$ ________

4.100 $18b - 20b$ ________

4.101 $5x - 6y + 4x$ ________

4.102 $10x - 3x + 4y - y$ ________

4.103 $7x + 2y - 5x + y$ ________

4.104 $5ab + 3ab - 2ab$ ________

4.105 $3ab + 4ab + 5$ ________

4.106 $3x + 2y - x - y - 1$ ________

4.107 $10xy + 3xy - 2xy$ ________

4.108 $6x^2y - 4xy^2$ ________

4.109 $13a - 12b + a - b$ ________

4.110 $5abc + 7abc$ ________

4.111 $7p^2q^2 - 3p^2q^2$ ________

Model 1: $3(x + 2) + 4x = 3x + 6 + 4x$ (distributive property)
$= 3x + 4x + 6$ (commutative property)
$= 7x + 6$

Model 2: $5(x - 2) + 3(x + 8) = 5x - 10 + 3x + 24$
$= 5x + 3x - 10 + 24$
$= 8x + 14$

Model 3: $7 + 2(x - 3) - 4(x - 1) = 7 + 2x - 6 - 4x + 4$
$= 2x + 7 - 6 - 4x + 4$
$= 2x + 1 - 4x + 4$
$= 2x - 4x + 1 + 4$
$= -2x + 5$

Simplify each of the following expressions.

4.112 $5(x - 2) + 6$ ________

4.113 $7(x + 6) - 4$ ________

4.114 $15(2 - x) + 17x$ ________

4.115 $7(x + 2) + 3(x + 1)$ ________

4.116 $8(x + 3) - 2x$ ________

4.117 $10(x - 5) + 20$ ________

4.118 $13 - 3(5 - 2x)$ ________

4.119 $10(x + 4) + 15(x + 1)$ ________

4.120 $13(x - 2) + 5(x + 1)$ ________

4.121 $8(2x + 2) + 5(3x - 1)$ ________

4.122 $7(9x - 3) - 4(2x - 1)$ ______

4.123 $10(7 - 2x) + 4(3 - x)$ ______

4.124 $2(x + 1) + 2(x + 2) + 3(x - 1)$ ______

4.125 $3(1 - 2x) + 2(x - 1) - 3(x - 4)$ ______

4.126 $2(x^2 - 1) + 3(x^2 + 1)$ ______

4.127 $6(x - 3) - 4(x + 1)$ ______

4.128 $4(5x - 6) - 4(2x + 1)$ ______

4.129 $4(6 - 2x) - 3(3 - 4x)$ ______

4.130 $20(1 - 2x) + 4(7 - 2x)$ ______

4.131 $7(x - 1) - 2(x + 1) + 3(x - 4)$ ______

4.132 $5(a - b) + 6(a + b) - 7(a - 2b)$ ______

4.133 $5(a^3 - 2) + 6(a^3 - 4)$ ______

Before you take this last Self Test, you may want to do one or more of these self checks.

1. ______ Read the objectives. Determine if you can do them.
2. ______ Restudy the material related to any objectives that you cannot do.
3. ______ Use the SQ3R study procedure to review the material:
   a. **S**can the sections.
   b. **Q**uestion yourself again (review the questions you wrote initially).
   c. **R**ead to answer your questions.
   d. **R**ecite the answers to yourself.
   e. **R**eview areas you didn't understand.
4. ______ Review all activities and Self Tests, writing a correct answer for each wrong answer.

SELF TEST 4

Complete these activities; a partial answer is incorrect (each answer, 4 points).

4.01 List all the elements of Set A if $A = \{x \mid x \text{ is an integer and } -6 \leq x < 0\}$ ______

4.02 Using the *rule* method, describe the set $\{18, 3, 9, 6, 21, 15, 24, 12\}$.

______

4.03 How many subsets does the set {10, 13, 1, 5, 6} have? ______

Given $A = \{a, e, i, o, u\}$ $B = \{a, l, g, e, b, r\}$ $C = \{m, a, t, h\}$

4.04 $A \cup B$ = ______

4.05 $A \cap B$ = ______

4.06 $B \cup C$ = ______

4.07 $B \cap C$ = ______

4.08 $A \cup C$ = ______

4.09 $A \cap C$ = ______

4.010 $A \cap B \cap C$ = ______

4.011 Evaluate $7 \cdot 5 + 12 \div 4$ ______

4.012 Evaluate $8 + 3 \cdot 4 \div 6$ ______

4.013 Given $F(x) = 2x^2 - 3$, find $F(-2)$ ______

4.014 Given $f(x) = 3x - 1$ and $g(x) = -x + 6$, find $f(-2) + g(5)$

______

Given $G = \{(-1, 7), (-8, 2), (0, 0), (6, 6)\}$

4.015 Domain of G = ______

4.016 Range of G = ______

4.017 What is the range set of $f(x) = x^2$ if x is all real numbers?

______

4.018 Given $h(p) = p^2 - 3p + q$, find $h(x)$. ______

4.019 Evaluate 3^4. ______

4.020 Write $(2x^2)$ without exponents. ______

4.021 Simplify $(3a^2)^3$. ______

4.022 Find $(-2x^3)$ when $x = -2$. ______

4.023 Evaluate $6^0 + 6^1 + 6^2$. ______

Write *true* or *false* (each answer, 1 point).

4.024 ________ $7 \in \{5, 2, 1, 3, 7\}$

4.025 ________ $8 \subset \{7, 8, 9\}$

4.026 ________ $\emptyset = \{0\}$

4.027 ________ If $A \subset B$ then $A \cap B = \emptyset$

4.028 ________ $B \cup \emptyset = B$

4.029 ________ $A \cap \emptyset = A$

4.030 ________ $3 + 5 = 5 + 3$ is an example of the commutative property for addition.

4.031 ________ $5(3 + 2) = 15 + 10$ is an example of the associative property for addition.

4.032 ________ If $x = 2$ and $2 = p$, then $x = p$.

4.033 ________ $a(b \cdot c) = c(a \cdot b)$.

4.034 ________ $\{(5, 1), (6, -2), (6, 3), (8, 1)\}$ is a relation.

4.035 ________ $\{(-1, 2), (0, 2), (5, 2)\}$ is a function.

4.036 ________ $(x + y)^0 = x + y$.

4.037 ________ $\frac{1}{5^{-2}} = 25$.

4.038 ________ $10^2 \cdot 10^3 = 100^5$.

Simplify the following expressions (each answer, 3 points).

4.039 $\frac{32a^3b^2}{8ab^2}$ ________

4.040 $48x^2(16x)^{-1}$ ________

4.041 $x^5 \cdot x^{-5} \cdot x^2$ ________

4.042 $3(x + 2) - 4x$ ________

4.043 $5(2x - 3) + 4(x + 1)$ ________

4.044 $5(x + y) + 3(x - y)$ ________

4.045 $3x^2 + 2x + 4 - x^2 + 5x$ ________

103/128

Score ________
Teacher Check ________
Initial Date

Before you take the LIFEPAC Test, you may want to do one or more of these self checks.

1. ______ Read the objectives. Determine if you can do them.
2. ______ Restudy the material related to any objectives that you cannot do.
3. ______ Use the SQ3R study procedure to review the material.
4. ______ Review all activities and Self Tests, and LIFEPAC Glossary.
5. ______ Restudy areas of weakness indicated by the last Self Test.

GLOSSARY

axiom. A statement accepted as true without proof.

closure. The condition that produces a unique element in the sum or product of two original elements.

domain. The set composed of first elements from a set of ordered-pair numbers.

element. One member of a set.

empty set. A set with no elements.

equal sets. Sets whose elements are identical.

equivalent sets. Sets that contain the same number of elements.

exponent. A small-sized number placed at the right of and above a symbol that serves to indicate the number of times the symbol appears as a factor.

exponential notation. A shorthand expression for repeated multiplication.

finite set. A set in which the number of elements is bounded by an interval.

function. A set of ordered-pair numbers such that for each first element there exists a unique second element.

infinite set. A set in which the number of elements is limitless.

integer. Any one of the numbers from negative infinity to positive infinity.

intersection of sets. A set whose elements are common to two other sets.

inverse of a function. A set of ordered-pair numbers in which the range and domain sets are interchanged.

natural numbers. Any one of the numbers from positive one to infinity.

operation. One of two mathematical procedures: addition or multiplication.

range. The set composed of second elements from a set of ordered-pair numbers.

relation. A set of ordered-pair numbers.

set. A collection of objects, concepts, or symbols.

subset. A set whose members are also members of a second set.

theorem. A statement requiring proof.

union of sets. A set whose elements appear in either of two other sets.

whole numbers. Any one of the numbers from zero to infinity.

Alpha Omega Publications®
804 N. 2nd Ave. E.
Rock Rapids, IA 51246-1759
800-622-3070
www.aop.com

MAT1101-May '07 Printing

ISBN 978-1-58095-461-7

9 781580 954617